MATH 33A Worksheet Week 7

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Exercise 1. Determine whether the following sets of vectors are *orthonormal* (orthogonal and unit length):

(a) $\begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}, \begin{bmatrix} -4/5 \\ 3/5 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(c) $\begin{bmatrix} 2/3 \\ -1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} -1/3 \\ 2/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 2/3 \\ 2/3 \\ -1/3 \end{bmatrix}$

(d) $\begin{bmatrix} a \\ a \end{bmatrix}, \begin{bmatrix} a \\ -b \end{bmatrix}, \begin{bmatrix} b \\ a \end{bmatrix}$ for $a, b \in \mathbb{R}$.

Exercise 2. Find a basis for $W^{\perp}$, where

$$W = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} \right\}$$

(Hint: How can we relate $W^{\perp}$ to subspaces where we know how to find a basis?)

Exercise 3. Find the orthogonal projection of $\begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$ onto the subspace

$$V = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Exercise 4. For each of the following vectors $\vec{v}$, find the decomposition $v^{||} + v^{\perp}$ with respect to the subspace

$$V = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

(a) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Exercise 5. Let $V = \text{span}\{\vec{v}_1, \dots, \vec{v}_k\}$ be a subspace of $\mathbb{R}^n$ where the vectors $\vec{v}_1, \dots, \vec{v}_k$ give an orthonormal basis for V.

(a) If $\vec{w} \in V$, show that $\text{proj}_V(\vec{w}) = \vec{w}$.

(b) If $\vec{w} \in V^\perp$, show that $\text{proj}_{V^\perp}(\vec{w}) = 0$.
